The Catholic University of America

ON THE AIRFLOW PAST A HEATED SPHERE

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL OF ARTS AND SCIENCES OF THE CATHOLIC UNIVERSITY OF AMERICA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

BY

IRVING HARTMANN, M.A., M.E.

THE CATHOLIC UNIVERSITY OF AMERICA
WASHINGTON, D. C.
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1. THE PROBLEM.

The experiments described in this paper relate to the flow caused by the motion of a solid in a fluid otherwise at rest. Ever since modern science came into existence this problem, on account of its particular importance in navigation and in other fields, has aroused the curiosity of the foremost minds. With the advent of aeronautics and the intensive study of aerodynamics the efforts to solve this problem have been greatly multiplied. The goal of the many theoretical investigations and experimental researches undertaken along this field has as yet not been reached. There exists to-day no satisfactory and rational explanation of what goes on in the fluid through which a solid moves, sufficient to enable us to predict the type of fluid motion in any particular instance, nor to compute the fluid forces associated therewith.

The present investigation falls in line with the many others which were undertaken to solve this problem. In common with them all, this paper constitutes but a step likely to advance us closer to the solution, rather than containing the solution itself.

2. THEORETICAL CONSIDERATIONS.

The most useful concept for the study of fluid motion along solid boundaries is the "boundary layer" introduced by L. Prandtl (1).[1] Prandtl was the first to appreciate the need for a particular classification of the regions of a moving fluid with respect to the type of its motion. It was well known before him that the fluid behind a moving solid is left in a state of more or less turbulent agitation, as opposed to the quiet and almost imperceptible motion near its front and

[1] Numbers in parentheses refer to bibliography at the end of paper.

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along its sides. Prandtl realized that this large region of apparently quiet motion should again be subdivided into two parts. In the first region, located at some distance from the boundary walls, the flow is practically unaffected by viscosity (potential flow). In the second region, directly adjacent to the walls, the viscosity forces as well as the inertia forces are of importance in determining the type of the flow.

This stratum adjacent to the surface has been called by Prandtl "boundary layer" (Grenzschicht). Making some plausible simplifying assumptions, he proceeded to investigate mathematically the flow inside the boundary layer. As far as it went the theory proved a success. Prandtl and his associates were able to explain rationally and in numerical detail some experimental facts unexplained before.

Unfortunately, the final outcome of these discoveries and investigations fell short of expectations. It developed that the application of Prandtl's differential equations was limited to a class of fluid motions which only rarely occur in technical problems. The equations are limited to steady laminar flows. Prandtl assumes the stability of this flow without further examination. Experience has shown however that in more technical problems the steady laminar motion is unstable and is replaced by a turbulent motion. This type of motion is stable but unsteady, just as the waving of a flag in stable equilibrium is unsteady. Both flag and flow perform an unsteady, more or less periodic motion, fluttering and oscillating, and this type of motion is persisting and takes the place of the simpler laminar flow within the boundary layer suggested by Prandtl.

A rational solution of the flow problem requires therefore a full understanding of the details of motion in an unsteady and turbulent boundary layer. This term has been retained from the beautiful researches of Prandtl, and it has been extended to signify the thin layer of fluid adjacent to the solid boundary, even when, in that region, the flow deviates to a much larger degree from a potential flow than was suggested by Prandtl. This turbulent layer has been mathematically investigated using assumptions based on experimental data, and some progress can be expected by pursuing this line of thought. However, no strictly rational treatment of the fluid motion in a turbulent boundary layer is possible as long as we are unacquainted with the reasons for the instability of the laminar boundary layer, and with the details of the turbulent motion taking the place of the laminar motion in the layer.

The present paper is devoted to the determination of the regions which govern the type of flow. Since the solid by its presence causes the motion, the regions very far from it cannot be expected to have much influence on the flow. Neither may the regions directly adjacent to the solid boundary be governing, since the walls, by completely shearing off the normal velocity component, and by damping the tangential velocity component, force these layers to move in a definite manner. The more specific question suggests itself, whether it is the inner or the outer parts of the boundary layer which govern the flow, or perhaps different laminae at different velocities. The following experiments have been undertaken to obtain an answer to this question. The answer may prove a valuable clue to the solution of the whole problem.

3. A NEW APPROACH.

The method adopted for obtaining information on this question is novel. It is known that equal values of equivalent Reynolds numbers indicate dynamic similarity of all conditions precedent, and hence are the necessary condition for the dynamic similarity of all conditions consequent. In other words, all non-dimensional coefficients describing the flow or its physical characteristics are functions of Reynolds number only. It is therefore a universal custom to study coefficients as functions of Reynolds number and to plot them accordingly. All points so plotted lie on a definite curve or a family of definite curves. For a fluid homogeneous in its physical properties, particularly in its kinematic viscosity, the Reynolds number of the set-up determines the type of flow coming into existence, for this Reynolds number is representative of all Reynolds numbers of the different portions of the flow.

More light on the mechanics of moving fluids is now sought by making the fluid surrounding the moving object non-homogeneous. The tests are so arranged that the kinematic viscosity of the fluid in the boundary layer is different from the kinematic viscosity without. This makes it possible to identify the flow-governing region by the Reynolds number characteristic of the resulting flow. The type of flow is observed by determining the drag coefficient of the solid in question. At first the relation between the Reynolds number and the drag coefficient in a homogeneous fluid is experimentally determined. One definite drag coefficient corresponding to each Reynolds number results. Next a drag measurement is made in a non-homogeneous fluid, and the drag coefficient computed. The Reynolds number corresponding to the same value of drag coefficient in the homogeneous fluid is then determined. From this Reynolds number, the flow velocity, and the model size, the kinematic viscosity is computed which corresponds to the homogeneous flow. The region or layer of the non-homogeneous fluid possessing this characteristic viscosity is finally ascertained, and this region is considered flow-governing or representative.

The solid used in the present experiments was a hollow brass sphere. By heating the sphere, the temperature of a thin layer of air surrounding it is raised, and its kinematic viscosity becomes greater than that of the colder air further removed from the sphere.

The thickness of this heated layer of air is of the same order of magnitude as the thickness of the boundary layer. This is true for turbulent as well as laminar flow, and follows from the close similarity of the mechanisms of heat diffusion and momentum diffusion. For laminar flow, the mechanism by which the transfer of momentum is brought about (in other words the cause of the frictional resistance responsible for the existence of the boundary layer) is the same as the mechanism by which the transfer of heat takes place. In both instances the exchange is brought about by molecular diffusion between adjacent laminae. Our knowledge concerning the mechanism controlling turbulent motion is at present still rather incomplete. However experiments of F. Elias (2) show that even for this motion the distribution of velocity and temperature are almost identical near the heated surface. This is the case, although very careful measurements recently made by A. Fage (3) indicate that when the general motion is eddying the conditions of flow even very near the solid surface are not the same as exist in a streamline flow, in other words no part of the boundary layer is laminar.

4. DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE.

The experiments were performed in a wind tunnel at the Catholic University grounds. The tunnel is of the Venturi type, and has a square cross-section of nine square feet in area. The air stream is generated by a four-bladed propeller driven by a 10 H.P. electric motor. The maximum air speed obtainable was 60 M.P.H., and the minimum constant air speed was about 25 M.P.H. The speed was varied by changing the R.P.M. of the motor. A wall plate located approximately three feet in front of the sphere and connected to an alcohol manometer was used for measuring the air speed. The wall plate had been calibrated against a pitot tube substituted for the sphere, so that the manometer readings indicated the speed of air in the region just approaching the sphere.

A hollow brass sphere of 5.01 inch outside diameter and 0.026 inch thickness was used for the experiments. It was equipped with a 660 watt electrical resistance heating unit. The electrical power input was measured by aid of an ammeter and voltmeter. The power input and sphere temperature were varied by means of a resistance in series with the heating coil. To measure the temperature of the sphere four copper-constantan thermocouples were connected to as many points inside the spherical shell, and to a Leeds and Northrup potentiometer. The temperature was sensibly constant over the entire surface of the sphere even at high air speeds, and all four thermocouples gave the same readings. The surface of the sphere was polished to reduce radiation losses.

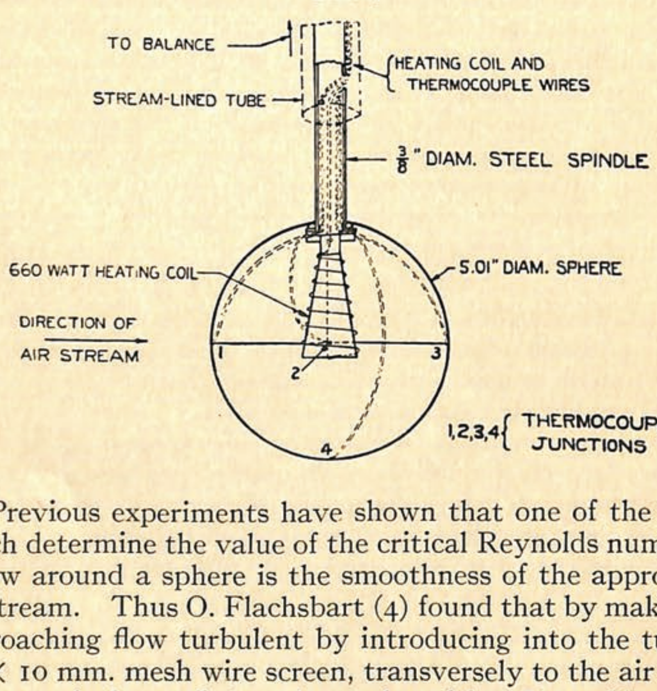
Figure 1 shows a cross-section of the sphere used in the tests.

The drag of the sphere was measured by means of a balance of the moment type. The scale beam of the balance is graduated in thousandths of a pound, and readings can be estimated to a tenth of the smallest graduations. The sphere was attached to the balance through a steel spindle of three-eighth inch diameter, which had been hollowed over part of its length, so that the thermocouple and resistance wires entered the sphere without affecting the air flow around it. To further prevent a disturbance of the flow a streamlined hollow tube was placed around the spindle up to a distance of two inches from the sphere.

During each test the room temperature and atmospheric pressure were observed.

It was originally intended to perform the tests in the so-called critical Reynolds number region, where the variation of the drag coefficient with Reynolds number is most pronounced. But the air speed which could be obtained in the tunnel was not sufficiently high to reach the critical Reynolds number for the set-up as it was first arranged.

FIG. 1.



Previous experiments have shown that one of the factors which determine the value of the critical Reynolds number for a flow around a sphere is the smoothness of the approaching air stream. Thus O. Flachsbart (4) found that by making the approaching flow turbulent by introducing into the tunnel a 10 × 10 mm. mesh wire screen, transversely to the air flow at 0.5 meter in front of the sphere, the critical Reynolds number of his set-up was reduced to about half its original value. In our experiments a similar method was adopted to reduce the critical Reynolds number.

In all, three series of experiments were performed. In the first there was no obstruction in the wind tunnel except a honeycomb at the entrance which remained there throughout all tests, the purpose of which is to break up large scale variations in the entering air flow. A second set of tests was performed with a 3/8″ × 3/8″ mesh wire screen placed 20 inches in front of the sphere and covering the entire tunnel section (36″ × 36″). The introduction of the wire screen reduced the available air speed. In order to remedy this, and at the same time to increase the turbulence slightly, a third set of tests was made with a 3/8″ × 3/8″ mesh wire screen stretched only across the central (18″ × 18″) part of the tunnel section 15 inches in front of sphere. The results of the three series of tests are given in separate tables on pages 601, 603, 605 and the curves plotted from these results are shown in separate figures on pages 607, 608, 609.

For each change in the tunnel the wall plate was recalibrated against a pitot tube. Before the tests a spindle correction curve was obtained in the usual manner. The values of spindle drag thus obtained were then subtracted from the total drag, so that the resistance values used in computing the drag coefficient represent only the drag of the sphere.

Two different procedures were followed in performing the experiments. At first, the sphere temperature was kept constant, while the sphere resistance was measured at various air speeds. This was done with the sphere at room temperature, as well as at a few higher temperatures. Following this, the air speed was kept constant while the sphere temperature was changed and the corresponding drag measured. In all cases it was necessary to wait a little before taking readings, in order to make certain that sphere temperature and air speed had assumed constant values. The points plotted from the observations in these two different ways are designated by different symbols in the curves that follow. All points obtained by the second method are shown by solid dots.

Measurements were also made to determine the heat losses from the sphere at various air speeds and several values of temperature difference between sphere and air. This required very little change in the set-up. Information on the subject is lacking and the results may have some direct bearing on the problem.

5. EXPERIMENTAL RESULTS.

For the purpose of clearer presentation of the subject we shall here introduce the term "local Reynolds number" as differentiated from the "common Reynolds number." While the latter is based on the kinematic viscosity of the fluid at large, which depends on the prevailing room temperature, the "local Reynolds number" is based on the kinematic viscosity of a certain region in question near the surface of the sphere. Unless specifically stated otherwise, the term "Reynolds number" will be implied to mean the "common Reynolds number."

The results of the drag measurements in the three sets of experiments are given in Tables I to III. The following notation is used in the tables:

A = Cross-sectional area of great circle of sphere,

V = Velocity of undisturbed air,

ρ = Density of air at prevailing room temperature,

R_C = Common Reynolds number,

ν_M = Kinematic viscosity of air at mean layer temperature (arithmetical mean of sphere temperature and room temperature),

ν_C = Kinematic viscosity of air at room temperature,

R_M = Local Reynolds number based on kinematic viscosity at mean layer temperature.

The values of drag coefficient and Reynolds number given in the tables were computed from the measurements of drag, speed, temperature, etc.

The Reynolds number is defined by $R = \frac{LV}{\nu}$, where L is diameter of sphere (5.01″), V has the meaning given above, and ν, the kinematic viscosity of air, is equal to the ratio of the coefficient of viscosity to the mass density.

The drag coefficient is defined by $C_D = \frac{\text{Drag}}{A V^2 \rho / 2}$, where the symbols have the meaning given above.

Using the values of drag coefficients and Reynolds numbers given in the tables the curves in Figs. 2 to 8 were plotted.

Figure 2 shows the effect of the wire screen on the flow around the unheated sphere. As may be seen the introduc-

TABLE I.

Drag Measurements without Wire Screen in Tunnel.

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
28.0	41.07	.260	.133	.515	98.9	31.5	31.5	.000173	98.9
32.0	46.93	.339	.168	.495	113.0	↓	↓	↓	113.0
36.0	52.80	.429	.204	.475	127.2				127.2
40.0	58.67	.530	.240	.455	141.3				141.3
44.0	64.53	.641	.278	.435	155.4				155.4
48.0	70.40	.763	.310	.405	169.6				169.6
52.0	76.28	.896	.337	.375	183.7				183.7
56.0	82.14	1.039	.360	.345	197.8				197.8
60.0	88.00	1.193	.383	.320	212.0				212.0
28.2	41.36	.264	.136	.520	99.6	31.5	31.5	.000173	99.6
32.1	47.08	.341	.169	.495	113.4	↓	↓	↓	113.4
36.0	52.80	.429	.205	.475	127.2				127.2
40.1	58.81	.533	.242	.455	141.6				141.6
42.0	61.60	.585	.259	.440	148.4				148.4
46.0	67.47	.701	.295	.420	162.5				162.5
50.1	73.49	.832	.324	.390	177.0				177.0
54.2	79.50	.973	.349	.360	191.5				191.5
28.1	41.21	.262	.137	.525	99.3	61.0	46.2	.000188	91.4
32.0	46.93	.339	.172	.505	113.0	↓	↓	↓	104.0
40.1	58.81	.533	.250	.470	141.6				130.7
44.2	64.82	.647	.292	.450	156.1				144.0
48.3	70.84	.774	.326	.420	170.6				157.0
52.2	76.57	.903	.353	.390	184.4				170.0
56.0	82.14	1.039	.371	.355	197.8				182.0
Atm. Press.: 760 mm. Hg.; Room Temp.: 26.0° C.; Air Density: .00229 lb./cu. ft.; ν_C: .0001675 ft.²/sec.									
32.0	46.93	.346	.174	.505	116.8	61.0	43.5	.000185	105.7
40.0	58.67	.540	.249	.460	146.0	↓	↓	↓	132.1
44.0	64.53	.653	.291	.445	160.6				145.3
48.0	70.40	.778	.326	.420	175.2				158.5
52.0	76.28	.913	.353	.385	189.8				171.8
56.0	82.14	1.059	.372	.350	204.4				185.0

TABLE I—*Continued.*

Manometer Reading.	Air Speed F.P.S.	$A V^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
30.0	44.00	.298	.155	.520	106.0	95.0	63.2	.000207	88.5
38.0	55.73	.478	.235	.490	134.2	↓	↓	↓	112.3
42.0	61.60	.585	.278	.475	148.4				124.0
46.1	67.61	.704	.318	.450	162.8				136.0
50.3	73.78	.838	.357	.425	177.7				148.2
54.1	79.36	.970	.386	.400	191.1				159.6
58.1	85.22	1.118	.418	.375	205.3				171.2
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
34.0	44.87	.383	.190	.495	120.0	43.0	37.2	.000179	116.1
34.0	44.87	.383	.192	.500	120.0	73.0	52.2	.000194	106.0
34.0	44.87	.383	.195	.510	120.0	95.0	63.2	.000207	100.5
38.0	55.73	.478	.227	.475	134.2	47.0	39.2	.000181	127.8
38.0	55.73	.478	.231	.485	134.2	73.0	52.2	.000194	118.7
38.0	55.73	.478	.236	.495	134.2	95.0	63.2	.000207	112.4
44.0	64.53	.641	.284	.445	155.4	49.5	40.5	.000182	148.6
44.0	64.53	.641	.290	.450	155.4	73.0	52.2	.000194	137.5
44.0	64.53	.641	.295	.460	155.4	95.0	63.2	.000207	130.0
50.0	73.34	.828	.330	.400	176.6	47.0	39.2	.000181	167.8
50.0	73.34	.828	.341	.410	176.6	73.0	52.2	.000194	156.1
50.0	73.34	.828	.355	.430	176.6	95.0	63.2	.000207	148.0
54.0	79.21	.966	.353	.365	191.0	45.0	38.2	.000180	182.5
54.0	79.21	.966	.371	.385	191.0	73.0	52.2	.000194	168.8
54.0	79.21	.966	.383	.395	191.0	95.0	63.2	.000207	159.5

tion of the screen into the tunnel causes the drag curve to be shifted toward smaller Reynolds numbers.

Figures 3 to 5 show the characteristic curves for the three different conditions of the approaching air stream. In each figure curves are given for the unheated sphere, as well as for the sphere at a few higher temperatures. The temperature of the sphere is indicated along each curve. The Reynolds numbers in these figures are common Reynolds numbers, based on the kinematic viscosity at the prevailing room temperature, irrespective of the sphere temperature. The room temperatures are given in each figure.

Figures 6 to 8 are plotted from values computed from the

TABLE II.

Drag Measurements with Wire Screen 20″ in Front of Sphere.

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
24.0	40.19	.254	.112	.440	100.3	25.5	25.5	.000167	100.3
28.0	46.93	.347	.141	.405	117.1				117.1
32.0	53.83	.456	.168	.370	134.3				134.3
36.0	60.43	.575	.190	.331	150.8				150.8
38.0	63.80	.641	.202	.315	159.2				159.2
40.0	67.47	.716	.214	.300	168.3				168.3
42.0	70.40	.780	.228	.290	175.6				175.6
44.0	74.51	.874	.247	.285	185.9	↓	↓	↓	185.9
22.0	36.67	.212	.098	.460	91.5	25.5	25.5	.000167	91.5
24.0	40.19	.254	.112	.440	100.3				100.3
26.0	43.41	.296	.127	.430	108.3				108.3
28.0	46.93	.347	.142	.410	117.1				117.1
30.0	50.60	.403	.155	.385	126.2				126.2
32.0	53.83	.456	.169	.370	134.3				134.3
34.0	57.20	.515	.180	.350	142.7				142.7
40.0	67.47	.716	.212	.295	168.3				168.3
42.0	70.40	.780	.226	.290	175.6	↓	↓	↓	175.6
22.0	36.67	.212	.099	.470	91.5	61.0	43.2	.000185	82.6
24.0	40.19	.254	.116	.455	100.3				90.5
26.0	43.41	.296	.130	.440	108.3				97.8
28.0	46.93	.347	.147	.425	117.1				105.7
30.0	50.60	.403	.161	.400	126.2				114.0
34.0	57.20	.515	.187	.365	142.7				128.8
36.0	60.43	.575	.197	.340	150.8				136.1
38.0	63.80	.641	.206	.320	159.2				143.7
40.0	67.47	.716	.218	.305	168.3				151.9
42.0	70.40	.780	.231	.295	175.6	↓	↓	↓	158.5
22.0	36.67	.212	.101	.475	91.5	84.0	54.8	.000197	77.6
24.0	40.19	.254	.119	.465	100.3				85.0
28.0	46.93	.347	.150	.430	117.1				99.3
30.0	50.60	.403	.165	.410	126.2				107.0
32.0	53.83	.456	.182	.400	134.3				113.9
34.0	57.20	.515	.194	.375	142.7				121.0
36.0	60.43	.575	.205	.355	150.8				127.8
38.0	63.80	.641	.217	.340	159.2				134.9
40.0	67.47	.716	.229	.320	168.3				142.7
42.0	70.40	.780	.241	.310	175.6	↓	↓	↓	148.9

TABLE II—*Continued.*

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
22.0	36.67	.212	.102	.480	91.5	106.0	65.8	.000209	73.1
24.0	40.19	.254	.120	.470	100.3				80.1
26.0	43.41	.296	.135	.455	108.3				86.6
28.0	46.93	.347	.152	.440	117.1				93.6
32.0	53.83	.456	.186	.410	134.3				107.3
34.0	57.20	.515	.200	.390	142.7				114.1
36.0	60.43	.575	.216	.375	150.8				120.5
38.0	63.80	.641	.226	.355	159.2				127.2
40.0	67.47	.716	.237	.330	168.3				134.5
42.0	70.40	.780	.248	.320	175.6	↓	↓	↓	140.4
24.0	40.19	.254	.115	.455	100.3	61.0	43.2	.000185	90.5
24.0	40.19	.254	.118	.465	100.3	84.0	54.8	.000197	85.0
32.0	53.83	.456	.174	.380	134.3	61.0	43.2	.000185	121.2
32.0	53.83	.456	.180	.395	134.3	84.0	54.8	.000197	113.9
32.0	53.83	.456	.186	.405	134.3	106.0	65.8	.000209	107.3
40.0	67.47	.716	.220	.305	168.3	61.0	43.2	.000185	151.9
40.0	67.47	.716	.229	.320	168.3	84.0	54.8	.000197	142.7
40.0	67.47	.716	.238	.332	168.3	106.0	65.8	.000209	134.5

same measurements as used in Figs. 3 to 5, but the Reynolds numbers here given are local Reynolds numbers based on the kinematic viscosity corresponding to the arithmetical mean of room and sphere temperature. This might also be called "mean layer temperature," assuming linear temperature variation in the boundary layer.

The results of the heat loss measurements at various air speeds and different screen positions, for a number of temperature differences between sphere and room, are plotted in Fig. 9. Although these measurements are not very exhaustive the curves seem to indicate that the more turbulent the flow reaching the sphere, the greater the quantity of heat which it carries away. This is really to be expected, since an eddying air stream means greater mixing of parts and reduces the thickness of the conducting film near the surface, thereby increasing the temperature gradient. Figure 9 is here inserted because the knowledge of the heat losses may prove important toward the solution of the final problem. They may also be used for the determination of the thickness of the boundary layer. The values of heat losses given in this figure include

TABLE III.

Drag Measurements with Screen 15" in Front of Sphere.

Manometer Reading	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$	Sphere Temp. °C.	Mean Layer Temp. °C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$
Atm. Press.: 758 mm. Hg.; Room Temp.: 26.0° C.; Air Density: .00229 lb./cu. ft.; ν_C: .0001675 ft.²/sec.									
26.0	39.60	.246	.104	.420	98.5	26.0	26.0	.0001675	98.5
28.0	42.53	.284	.116	.410	105.8				105.8
30.0	45.47	.324	.128	.395	113.1				113.1
32.0	48.40	.368	.140	.380	120.4				120.4
34.0	51.33	.413	.150	.365	127.7				127.7
36.0	54.56	.467	.161	.345	135.7				135.7
38.0	57.79	.524	.171	.325	143.8				143.8
40.0	61.31	.590	.182	.310	152.5				152.5
42.0	64.24	.648	.194	.300	159.8				159.8
44.0	67.17	.708	.204	.290	164.6				164.6
48.0	73.48	.847	.239	.280	182.8				182.8
50.0	76.71	.923	.258	.280	190.9	↓	↓	↓	190.9
26.0	39.60	.246	.104	.420	98.5	26.0	26.0	.0001675	98.5
30.0	45.47	.324	.127	.390	113.1				113.1
34.0	51.33	.413	.150	.360	127.7				127.7
38.0	57.79	.524	.170	.325	143.8				143.8
42.0	64.24	.648	.192	.295	159.8				159.8
46.0	70.25	.774	.221	.285	174.8				174.8
50.0	76.71	.923	.256	.280	190.9	↓	↓	↓	190.9
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.0° C.; Air Density: .00230 lb./cu. ft.; ν_C: .0001665 ft.²/sec.									
26.0	39.60	.247	.107	.430	99.1	61.0	43.0	.000185	89.2
30.0	45.47	.326	.132	.405	113.8				102.4
34.0	51.33	.415	.155	.375	128.5				115.6
38.0	57.79	.526	.175	.335	144.6				130.1
42.0	64.24	.650	.197	.305	160.8				144.7
46.0	70.25	.777	.224	.290	175.8				158.2
50.0	76.71	.926	.260	.280	192.0	↓	↓	↓	172.8
26.0	39.60	.247	.110	.445	99.1	84.0	54.5	.000197	83.8
30.0	45.47	.326	.137	.420	113.8				96.2
34.0	51.33	.415	.160	.385	128.5				108.6
38.0	57.79	.526	.181	.345	144.6				122.2
42.0	64.24	.650	.202	.310	160.8				135.9
46.0	70.25	.777	.226	.290	175.8				148.6
50.0	76.71	.926	.262	.285	192.0	↓	↓	↓	162.2

TABLE III—*Continued.*

Manometer Reading	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C\times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M\times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
26.0	39.60	.247	.113	.455	98.8	106.0	65.8	.000209	79.0
30.0	45.47	.326	.142	.435	113.4	↓	↓	↓	90.7
34.0	51.33	.415	.166	.400	128.1	↓	↓	↓	102.4
38.0	57.79	.526	.190	.362	144.2	↓	↓	↓	115.2
42.0	64.24	.650	.210	.325	160.3	↓	↓	↓	128.1
46.0	70.25	.777	.232	.300	175.3	↓	↓	↓	140.1
50.0	76.71	.926	.268	.290	191.4	↓	↓	↓	153.0
26.0	39.60	.247	.107	.435	98.8	61.0	43.2	.000185	89.2
26.0	39.60	.247	.110	.445	98.8	84.0	54.8	.000197	83.8
26.0	39.60	.247	.113	.460	98.8	106.0	65.8	.000209	79.0
34.0	51.33	.415	.156	.375	128.1	61.0	43.2	.000185	115.6
34.0	51.33	.415	.160	.385	128.1	84.0	54.8	.000197	108.6
34.0	51.33	.415	.166	.400	128.1	106.0	65.8	.000209	102.4
42.0	64.24	.650	.197	.305	160.3	61.0	43.2	.000185	144.7
42.0	64.24	.650	.203	.315	160.3	84.0	54.8	.000197	135.9
42.0	64.24	.650	.211	.325	160.3	106.0	65.8	.000209	128.1
50.0	76.71	.926	.261	.280	191.4	61.0	43.2	.000185	172.8
50.0	76.71	.926	.264	.285	191.4	84.0	54.8	.000197	162.2
50.0	76.71	.926	.269	.290	191.4	106.0	65.8	.000209	153.0

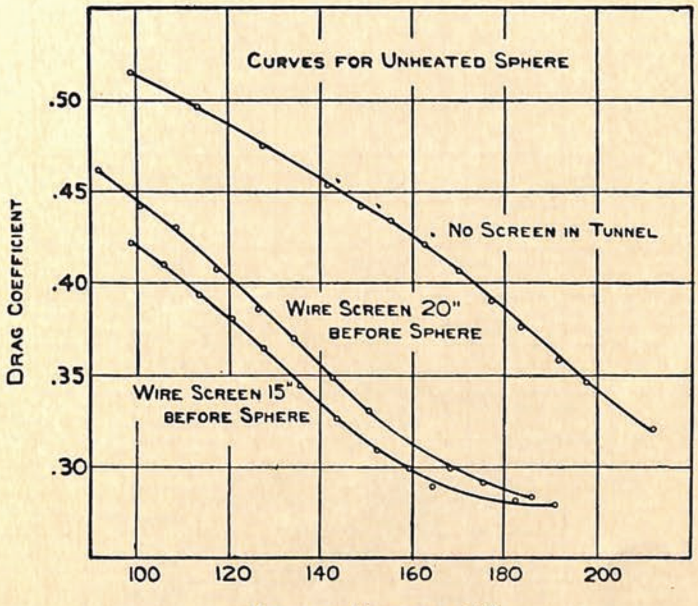
radiation as well as convection losses, but the former constitute only a small percentage of the total heat flow, amounting to less than 15 per cent even at the maximum temperature difference investigated.

6. DISCUSSION OF RESULTS.

As may be seen from Fig. 2 the introduction of the wire screen in front of the sphere served its intended purpose; namely, it caused a shift of the drag curve toward smaller Reynolds numbers. This is in accordance with the findings of other investigators (4). Moving the screen from 20 inches to a position 15 inches in front of the sphere did not change the flow greatly.

In Figs. 3 to 5 the effect of heating of the sphere is clearly seen. The curves in Fig. 5 might be interpreted as representing a region to the right of that covered by Fig. 3, while the curves in Fig. 4 fit in somewhere between these two sets. The curves show that raising the temperature of the sphere above that of the approaching air definitely shifts the drag curve. This change must be attributed to the changed properties of the boundary layer, since as we have seen, the heat flowing from the sphere affects only this thin layer, disregarding the relatively unimportant effect of the radiant heat. Our curves show that when the sphere is heated the drag coef-

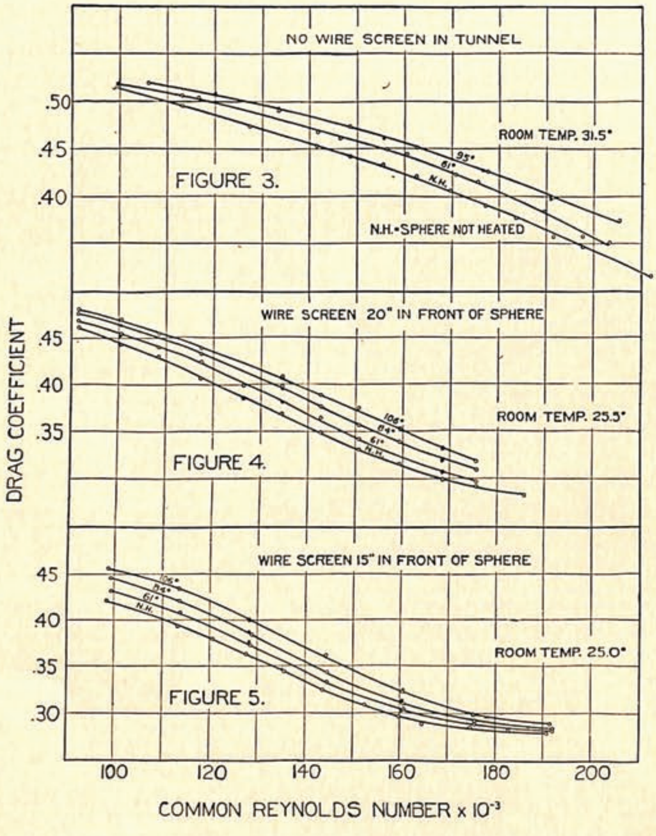
FIG. 2.



ficient at a given common Reynolds number is increased. But this is just the opposite of what happens when the air stream around the sphere is made more turbulent, as may be seen from Fig. 2. So we may say that as a result of the heat passing from the sphere into the fluid the flow becomes less turbulent, thus causing the point of separation of the boundary layer from the surface to move forward and the drag to increase. (See reference (5) for a discussion of the point of separation.)

In order to understand why increased temperature of the boundary layer should cause the flow to become smoother, we need only remember that increased temperature means increased air viscosity and decreased air density, and both

FIGS. 3, 4 AND 5.



these changes contribute toward increasing the kinematic viscosity and hence are conducive to smoother flow.

We can explain the shifting of the drag curves caused by the heating of the sphere in a more general way. On the basis of the principle of dynamic similarity, all points in the drag coefficient—Reynolds number diagram should lie on a single curve, provided that the Reynolds number values

FIGS. 6, 7 AND 8.

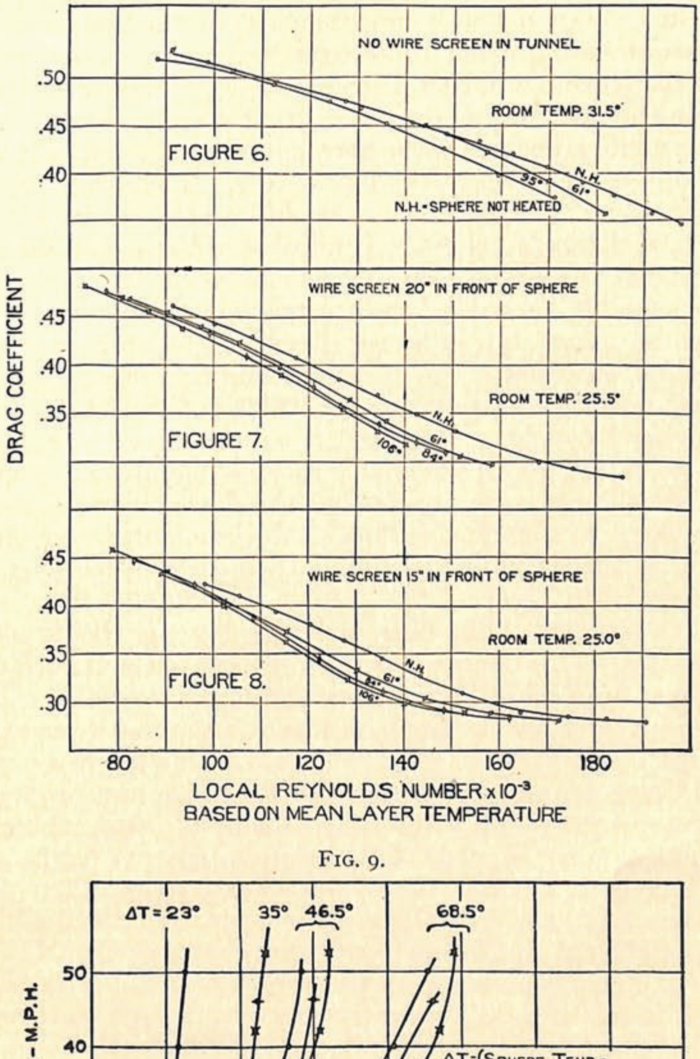
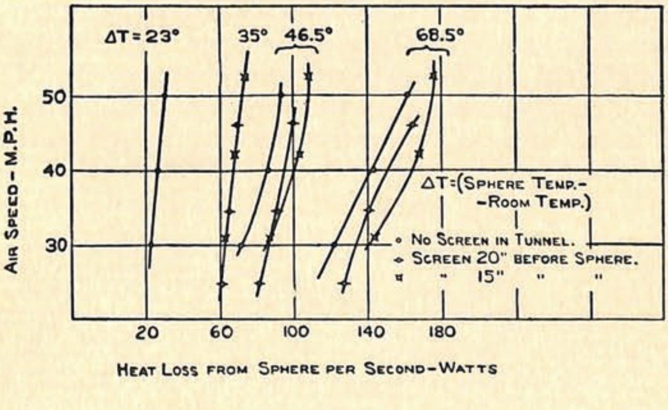


FIG. 9.



used are representative of the flow. Obviously then for the present case of a non-homogeneous fluid the common Reynolds numbers are not representative, but there exist some local Reynolds numbers corresponding to certain laminae within the heated boundary layer that are representative.

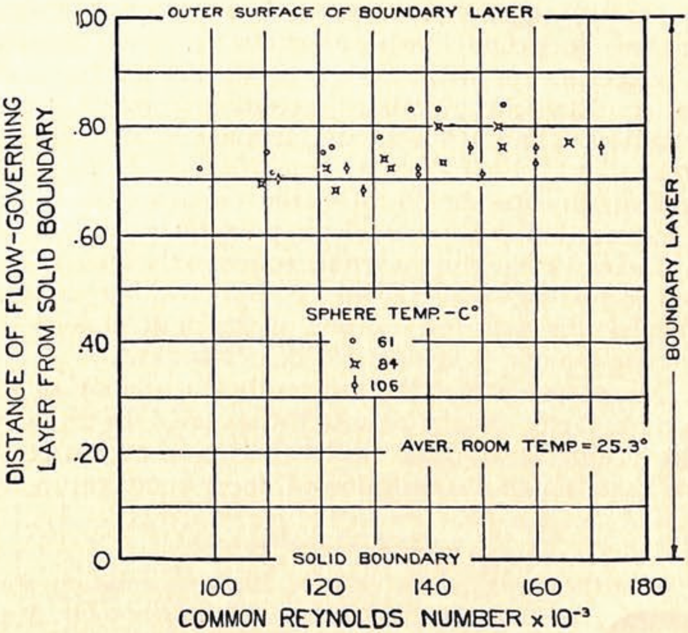
As a first step toward the determination of the flow-governing laminae we have plotted Figs. 6 to 8. The Reynolds numbers in these are based on the mean layer temperatures. In Fig. 6, which represents the conditions without any screen in the tunnel, it may be seen that at low Reynolds numbers the curves for the heated sphere nearly overlap the curve for the unheated sphere, but at higher speeds they branch to the left. This indicates that at low Reynolds numbers it is a layer near the center of the boundary layer which governs the flow, but at higher Reynolds numbers it is a layer closer to the outer film boundary which governs. In Figs. 7 and 8 the trend is similar. These curves show that the flow-governing layer is even closer to the outer surface of the boundary layer for the flow with the screen in the tunnel than for the smoother flow represented in Fig. 6.

The position of the flow-governing layer is clearly shown in Fig. 10. This figure gives the distances of the flow-governing layer from the solid boundary within the Reynolds number range investigated. The values were computed from curves in Figs. 4 and 5. This was done by selecting the layer whose local Reynolds number equals the Reynolds number for the unheated sphere at a given drag coefficient. In making this computation use is made of the relation between temperature and kinematic viscosity of air, and a linear temperature variation assumed across the boundary layer. The figure shows that in general the flow-governing layer moves slightly away from the solid boundary as the Reynolds number increases, and it moves toward the solid surface as the sphere temperature is raised.

7. CONCLUSIONS.

The foregoing experiments confirm the proposition that the properties of the boundary layer around a solid in a fluid moving relative to it are of great importance in determining the type of the flow. They show that within the Reynolds number range investigated (100,000–170,000), the flow around the sphere is governed by the properties of a lamina within the boundary layer, approximately seventy-five-hundredths of the boundary layer thickness away from the solid surface. The distance of this lamina varies slightly with the Reynolds number. It is closer to the surface for small Reynolds numbers, and moves farther out as the Reynolds number increases.

FIG. 10.



The position of the flow-governing layer seems to depend to a slight degree on the temperature of the sphere, and also on the relative turbulence of the approaching flow.

8. CONCLUDING REMARKS. SUGGESTIONS FOR FURTHER EXPERIMENTS.

The experiments described in the foregoing pages were carried out over a limited range of Reynolds numbers. They were also confined to a sphere at temperatures higher than the air stream. It is difficult to say whether the effects of the boundary layer, at Reynolds number beyond the range investigated, are of the same order as here found; whether in fact they are of any importance at all once the flow is quite turbulent. It was pointed out at the beginning of this paper that a knowledge of the causes for the instability of the laminar flow in the boundary layer is essential for the solution of the problem. Our experiments show that an outward diffusion of heat energy serves to stabilize the existing flow. Possibly by cooling of the solid greater instability will be brought about and turbulence promoted.

To answer these questions and to obtain further data along these lines the following experiments should be suitable:

1. Experiments with heated solids should be made in regions of Reynolds numbers extending beyond those here investigated. The tests should include solids other than spheres.

2. Experiments should be made on a sphere cooled to temperatures below those of the approaching air.

3. Instead of heating a whole sphere only a circular zone normal to the air stream should be heated. By varying the location of this strip the position of the most important section of the boundary layer could be determined.

4. Measurements of the temperature distribution should be made in the boundary layer of the sphere. In this way the flow-governing layer could be located more accurately than can be done on the assumption of linear temperature variation.

9. ACKNOWLEDGMENT.

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